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RECONSTITUTION OF THE RE-ENTRY INTO THE ATMOSPHERE OF A MISSILE, BASED ON DATA MEASURED IN FLIGHT **/1

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A method for calculating the probable trajectory of a missile on re-entry, based on data recorded in flight (accelerometer readings, drag coefficient, specific heat ratios, static pressure, dynamic pressure, etc.) is described, with graphs of the telemetered data. The calculation is performed on a DC analog computer with recording of the data on a varyplotter, permitting a rapid estimate and computation of the probable trajectory. Author

1. Introduction /2

The reconstruction of the trajectory of a missile is generally done on the ground by optical and radioelectronic observations (classical trajectography processes). When these specific means fail or else if the obtained data are to be verified, certain trajectory elements can be determined on the basis of measurements made on board. The computation method used in this case expresses the compatibility of various quantities measured in flight with the laws of mechanics. The purpose of this paper is to demonstrate the process by which this method, using a direct-current analog computer, has been applied to a specific case of a missile re-entering the atmosphere.

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** Numbers in the margin indicate pagination in the original foreign text.

2. Notations

j_T = Indication of the longitudinal accelerometer;

g = Acceleration of gravity;

m = Mass of the missile;

M = Mach number;

p_0 = Static pressure;

p_c = Body-base pressure;

p'_i = Impact pressure;

t = Time;

S_c = Surface of the body;

V = Missile velocity;

X = Characteristic coefficient of the net drag upstream of the body;

Z = Altitude;

γ = Specific heat ratio of the air;

ω = Mean inclination;

$\delta\omega$ = Deviation with respect to the mean inclination.

3. Problem Complex /3

Let us assume that the measurements made on board a missile, during re-entry into the atmosphere, have been transmitted to the ground only over two brief intervals of time (Fig.1) and that the classical trajectography processes have failed. Thus, the problem consists in determining, at any instant, the velocity, the altitude, and the inclination of the trajectory from collected data. These latter are numerous and we will give below only those that are necessary for reconstruction of the wanted trajectory elements:

3.1 The measurements of incidence, angular velocity, and transverse acceleration permit to confirm that the missile is correctly "re-aligned" during the period of interest (i.e., the velocity vector and the longitudinal axis of the missile practically coincide).

3.2 The important data for reconstruction of the trajectory elements are as follows:

- indication of the longitudinal accelerometer j_T (Fig.2);
- measurement of the impact pressure p'_i (Fig.3);
- measurement of the body-base pressure p_c.

3.3 The measurements of pressure and temperature, obtained by atmospheric radiosonde observations before and after blast-off, are introduced into the calculations.

4. Description of the Computation Method

The computational method used for determining certain trajectory elements from data collected during flight comprises the following stages:

a) Integration of the equations of motion on the basis of arbitrary initial conditions, taking the measurement or the calculation of the longitudinal accelerometer readings j_T into consideration. This phase also includes the calculation of the impact pressure p'_i at each point of the trajectory (Sect.4.1).

b) Comparison of the calculated and measured curves for $p'_i(t)$, followed by determination of the shape of the probable trajectory, based on a given criterion (Sect.4.2).

4.1 Integration of the Equations of Motion

As indicated above, the missile can be considered "re-aligned" at the instant at which the measured data are received; under these conditions, assuming the trajectory to be plane, the equations of motion fed to the DC analog computer, have the following form:

$$\frac{dV}{dt} = j_T + g \sin \varpi + g \cos \varpi \, \delta\varpi \tag{1}$$

$$\frac{d(\delta\varpi)}{dt} = \frac{1}{V}\left(g \cos \varpi - g \sin \varpi \, \delta\varpi\right) \tag{2}$$

$$\frac{dZ}{dt} = -V\left(\sin \varpi + \cos \varpi \, \delta\varpi\right) \tag{3}$$

4.1.1 Definition of a Calculated Trajectory

Each calculated trajectory is defined by three parameters which represent, at the instant t_3 (beginning of the second period of measurement), the following quantities (initial conditions of integration):

Altitude Z_3

Velocity V_3

Inclination of trajectory ϖ_3.

4.1.2 Determination of the j_T Reading of the Longitudinal Accelerometer

During the time intervals (t_1, t_2) and (t_3, t_4), the retained value of j_T is obtained from flight data (Fig.2). This value is introduced into the calculation by means of a function converter.

In the absence of measurements (period t_2, t_3), the value of j_T is calcu-

lated at each instant by an extrapolation from the law of aerodynamic drag, as a function of the Mach number. This extrapolation is sufficient for performing the integration of the equations of motion, since no propulsive unit is in operation at that moment. The expression derived for j_T has the following form:

$$\dot{j}_T = -\frac{X}{m}\frac{\gamma}{2} p_0 F(M) - \frac{S_c}{m} p_0 \left(1 - \frac{p_c}{p_0}\right) \qquad (4)$$

The first part of this expression [eq.(4)] represents the net drag upstream of the body while the second portion corresponds to the drag of the body base. /5

The terms m, γ, and Sc which are known characteristics either of the missile or of the air are considered as constant.

The obtained radiosonde data define the static pressure p_0 and the velocity of sound as a function of the altitude: From this and by means of function translators, the analog computer calculates the static pressures p_0 as well as the Mach number M at any instant.

The function F(M) is obtained from an analysis of wind-tunnel tests and, in the domain under study, can be represented by the following expansion

$$F(M) = 0.544 + 1.105\, M + 0.920\, M^2 \qquad (5)$$

The coefficients X and p_c/p_0, considered as characteristic constants of each calculated trajectory, are determined at the instant t_3 as follows:

a) The measurement, in flight, of the body-base pressure connected with the altitude data at the instant t_3 (and thus the corresponding static pressure) defines the ratio p_c/p_0.

b) After the ratio p_c/p_0 is defined, the coefficient X is adjusted in such a manner that the values of j_T, both calculated and measured,

will be identical at the instant t_3.

4.1.3 Calculation of the Impact Pressure p'_i

In accordance with the method described in Sections 4.1.1 and 4.1.2, the various elements of a trajectory (altitude, velocity, inclination) are calculated by integrating the equations of motion within the time interval (t_1, t_4). On the basis of combinations of these elements (Mach number, static pressure), the impact pressure, which is assumed to have been measured on board the missile, is determined by the following relation:

$$p_i'(t) = p_0\, G(M) \tag{6}$$

The characteristic function G(M) of the shock phenomena in the air is written into a function translator.

4.1.4 Computational Procedure

The various phases of the computation of a trajectory, defined by the initial conditions V_3, Z_3, ω_3, are as follows (Fig.4):

a) The equations of motion are integrated from t_3 to t_4 (measured value of j_T). /6

b) Based on the same initial conditions defined in item a), the equations of motion are integrated from t_3 to t_2 (decreasing time) by making use of the calculated value of j_T. At the instant t_2, a commutation permits to substitute - from t_2 to t_1 - the calculated value of j_T by the measured value of j_T.

4.2 Comparison of the Curves giving the Calculated and Measured Pressure $p'_i(t)$

The computational process is used for various values of V_3, Z_3, ω_3. The

most probable values of velocity, altitude, and inclination at the instant t_3 will be obtained from a comparison of the curves for calculated and measured $p'_1(t)$. The criterion derived for choice of the solution is defined as follows: The time integral of the mean-square deviation between the curves for calculated and measured $p'_1(t)$ must be minimal. So that the obtained result will be significant, it is necessary to make certain that, in a domain where V_3, Z_3, ω_3 vary greatly, the obtained solution is unique, i.e., that the time integral of the mean-square deviations then presents a distinct minimum.

Figure 5 gives an example for the calculations used in this verification. The inclination ω_3 is fixed at 90°; each curve $p'_1(t)$ corresponds to an altitude Z_3 (2000 m, 6000 m, 10,000 m), where the value of V_3 is so adjusted as to make the calculated and measured values of p'_1 coincide at the instant t_3: The observed regular deformation permits to assume that, within the covered range, the solution is unique.

5. Results Obtained

The suggested optimization method leads to adoption of the following initial values:

$$V_3 = 1050 \text{ m/sec}$$

$$Z_3 = 4800 \text{ m}$$

$$\omega_3 = 90^\circ.$$

The accuracy of the results (Figs.6, 7, 8) is difficult to estimate; a direct analysis is quite time-consuming and the corresponding computation would have to take the probable instrument errors in flight into consideration. Thus, it seems simpler, within the frame of investigating the problem by means of an analog computer, to use various computed trajectories as a basis for estimating

the possible instrument errors with respect to p'_1 for the case of modifications in V_3 and Z_3 in the vicinity of the solution used.

The error produced by the analog computer is negligible: The various elements used (function translator, etc.) actually have an accuracy quite superior to that of the board instruments. /7

In the case in question here (see Fig.9), a variation in V_3 of 70 m/sec, within the measuring time interval, will lead to a mean-square value of 1000 mb. A variation in altitude of Z_3 of 2300 m gives the same result.

For comparison, in the case of the solution used here, the maximum deviation between the calculated values of p'_1 and the measured values of p'_1 is 450 mb, while the mean-square deviation during the period of measurement is 50 mb. These differing data which depend on the quality of the scoops permit an estimate of the accuracy of the results obtained.

6. Conclusions

The computational means used for applying the suggested method can be of the analog or arithmetic type.

In the case discussed here, the use of a DC analog computer was found highly practical. In fact, the sought accuracy is entirely compatible with that of the various elements of the analog computer. In addition, the writing of the various functions involved (law of j_T, etc.), the transfer from one type of computation to another (see computational procedure), and the adjustment of certain coefficients (p_c/p_0, X) are relatively simple operations here. All these possibilities, together with a direct visual display of the obtained results (recorded on varyplotters) permit a rapid estimate and calculation of the probable trajectory of the missile on the basis of telemetered elements.

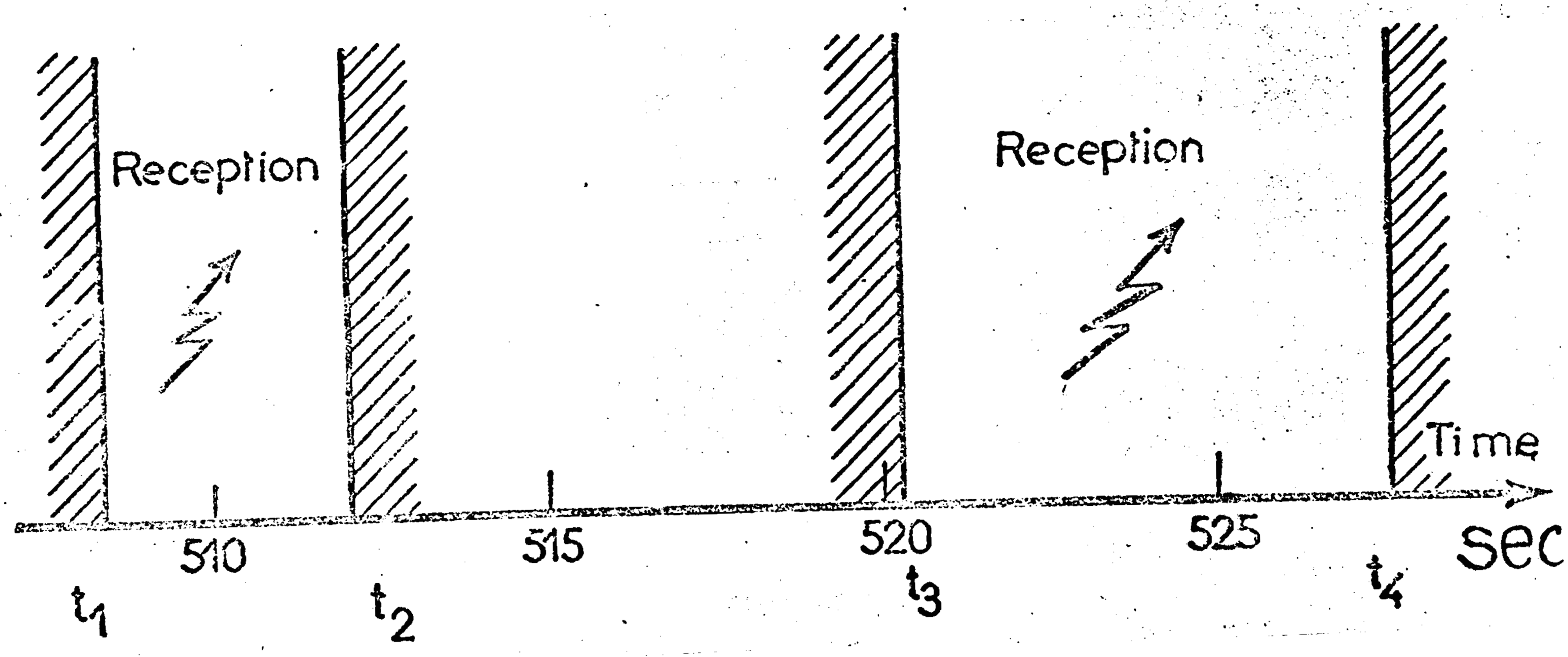

Fig.1 Reception of Telemetered Data

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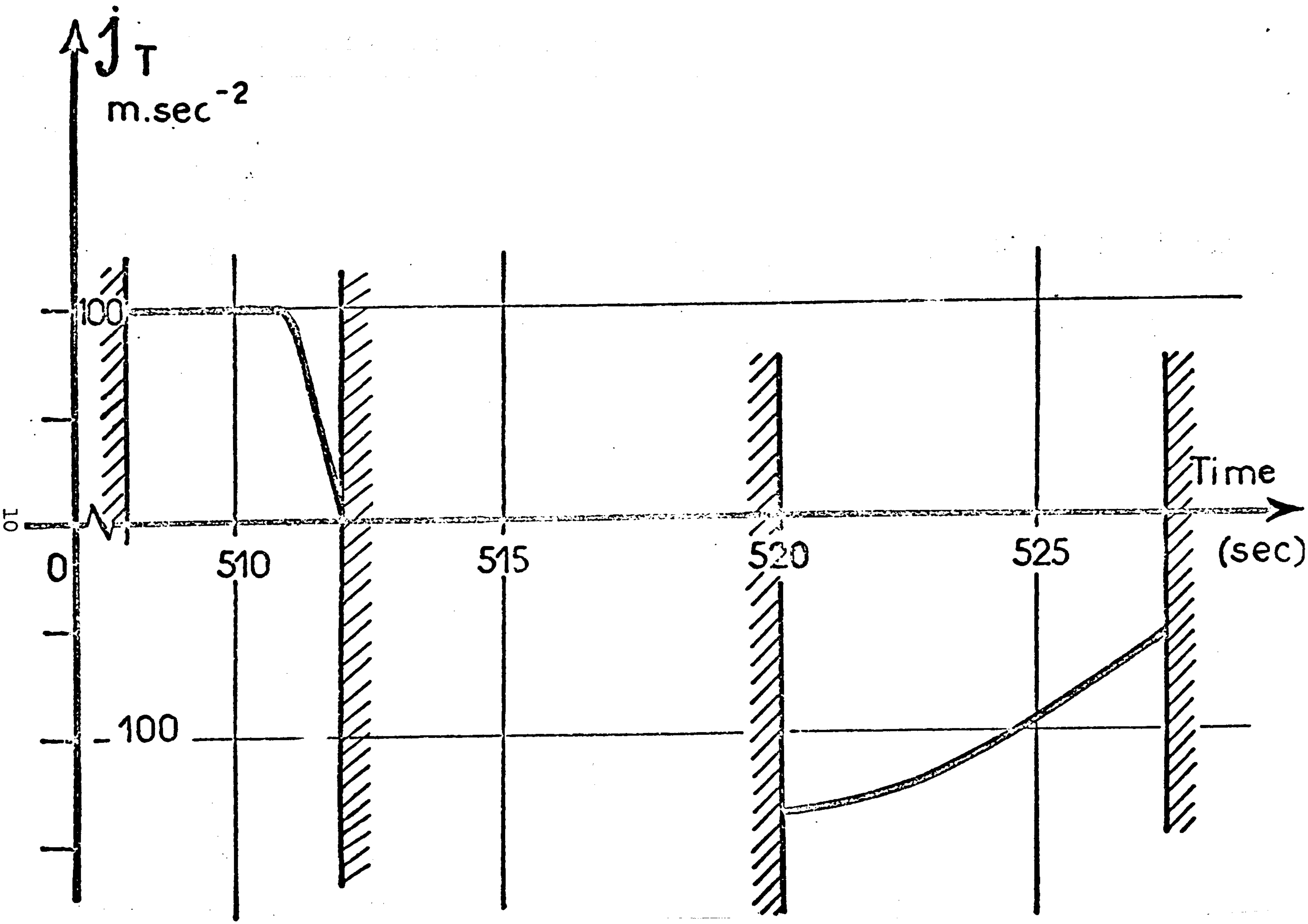

Fig.2 Longitudinal Accelerometer Readings

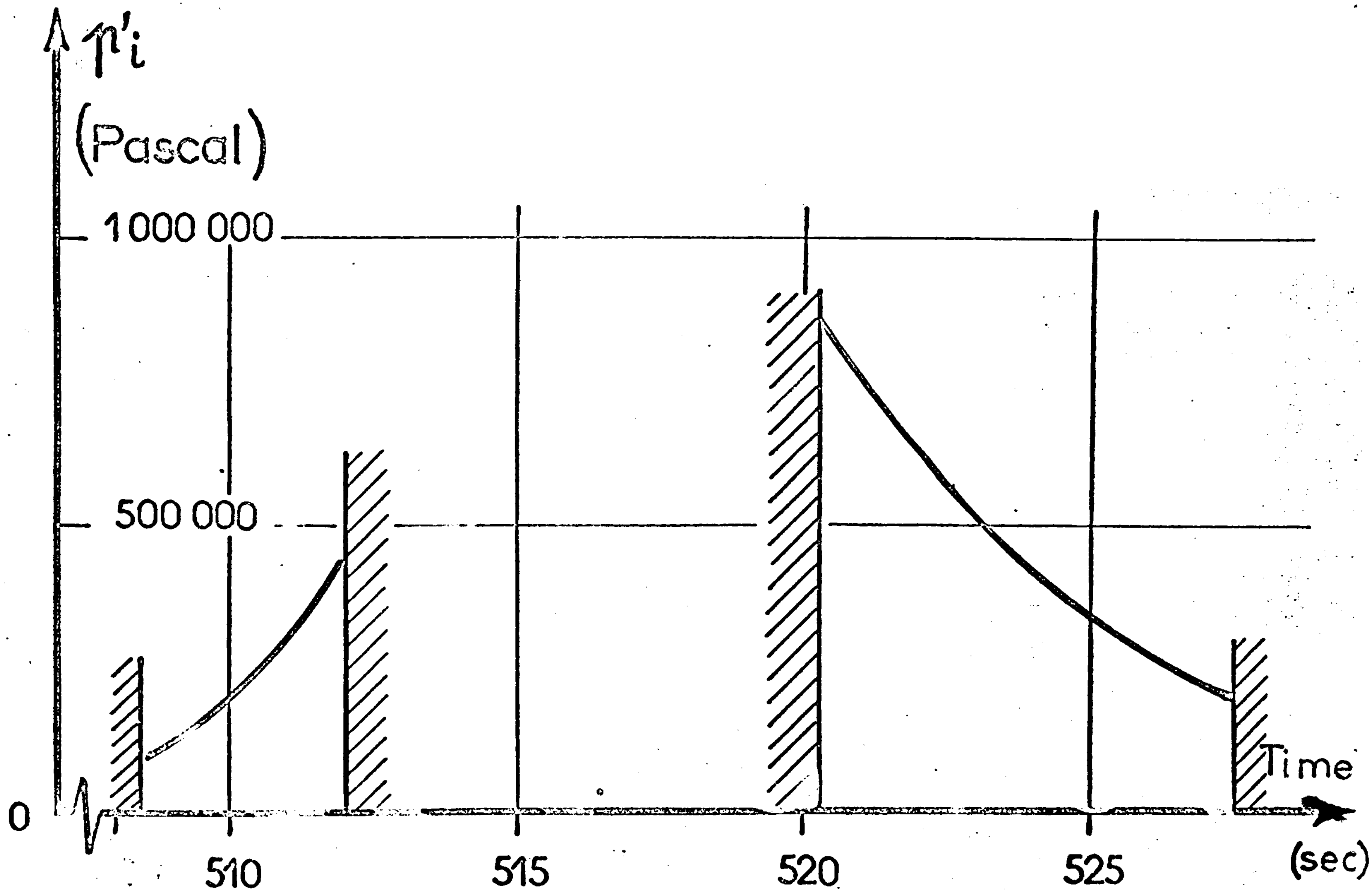

Fig.3 Measurement of the Impact Pressure

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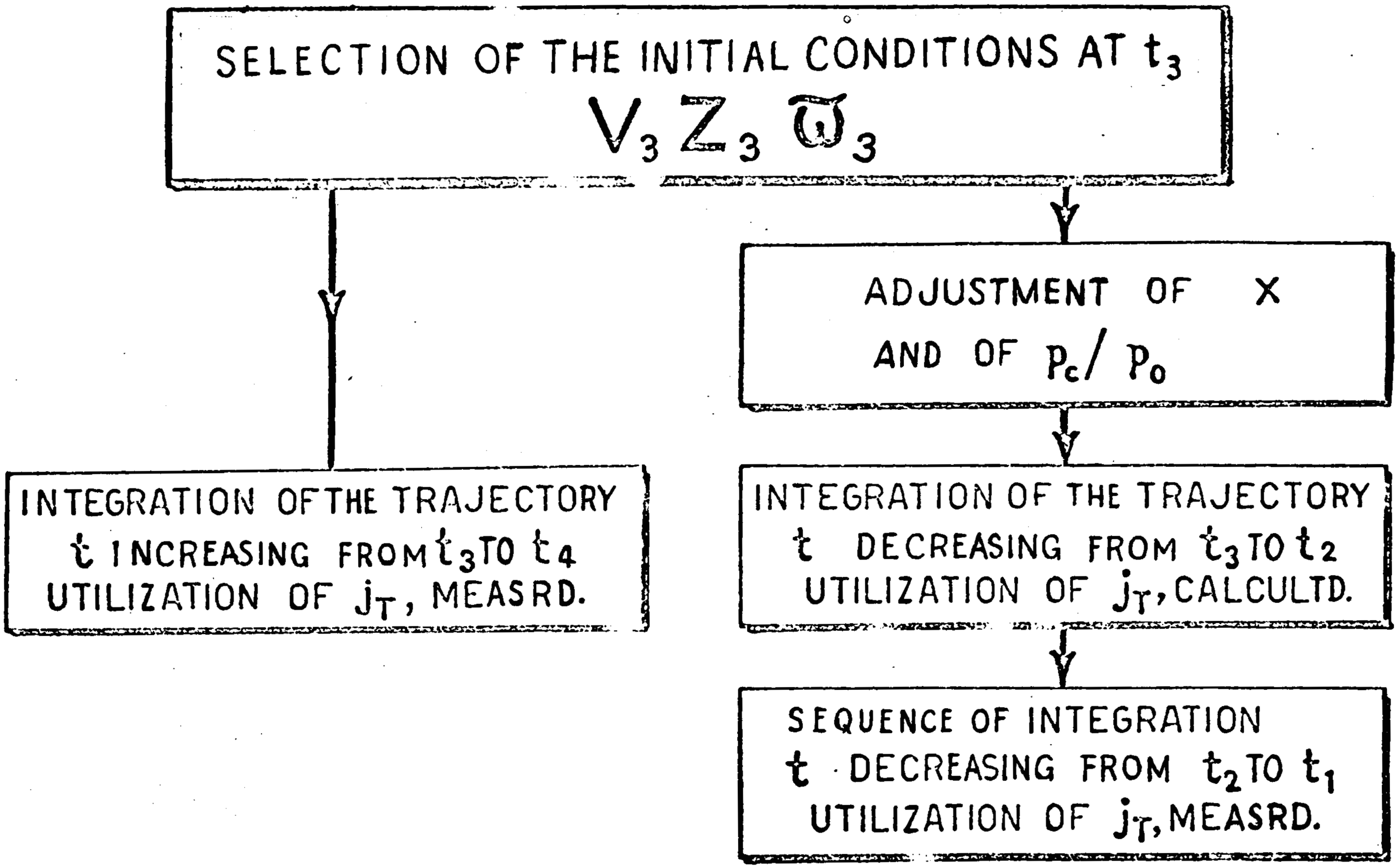

Fig.4 Computation Procedure

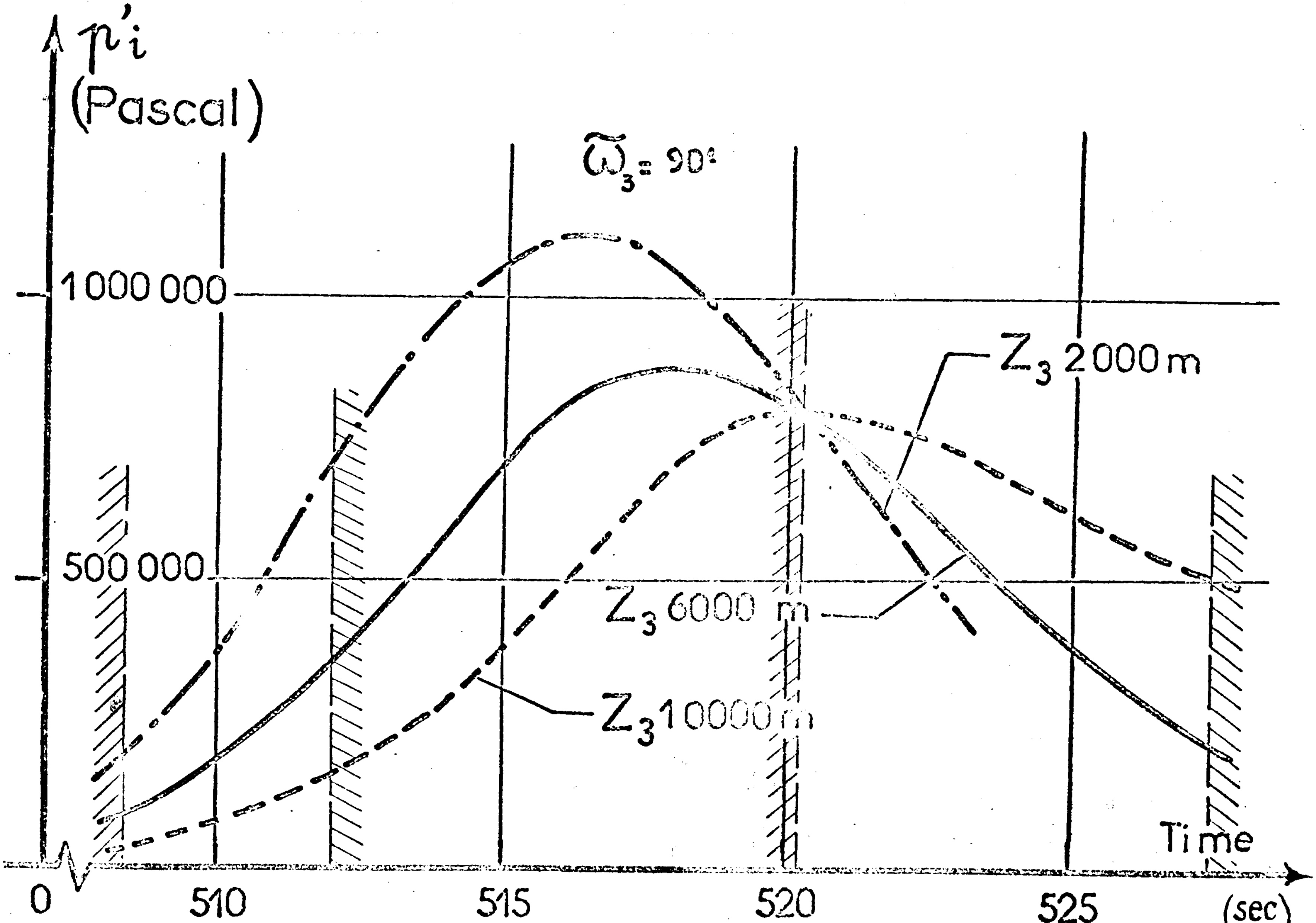

Fig.5 Calculation of the Impact Pressure; Sweep of the Value of Z_3

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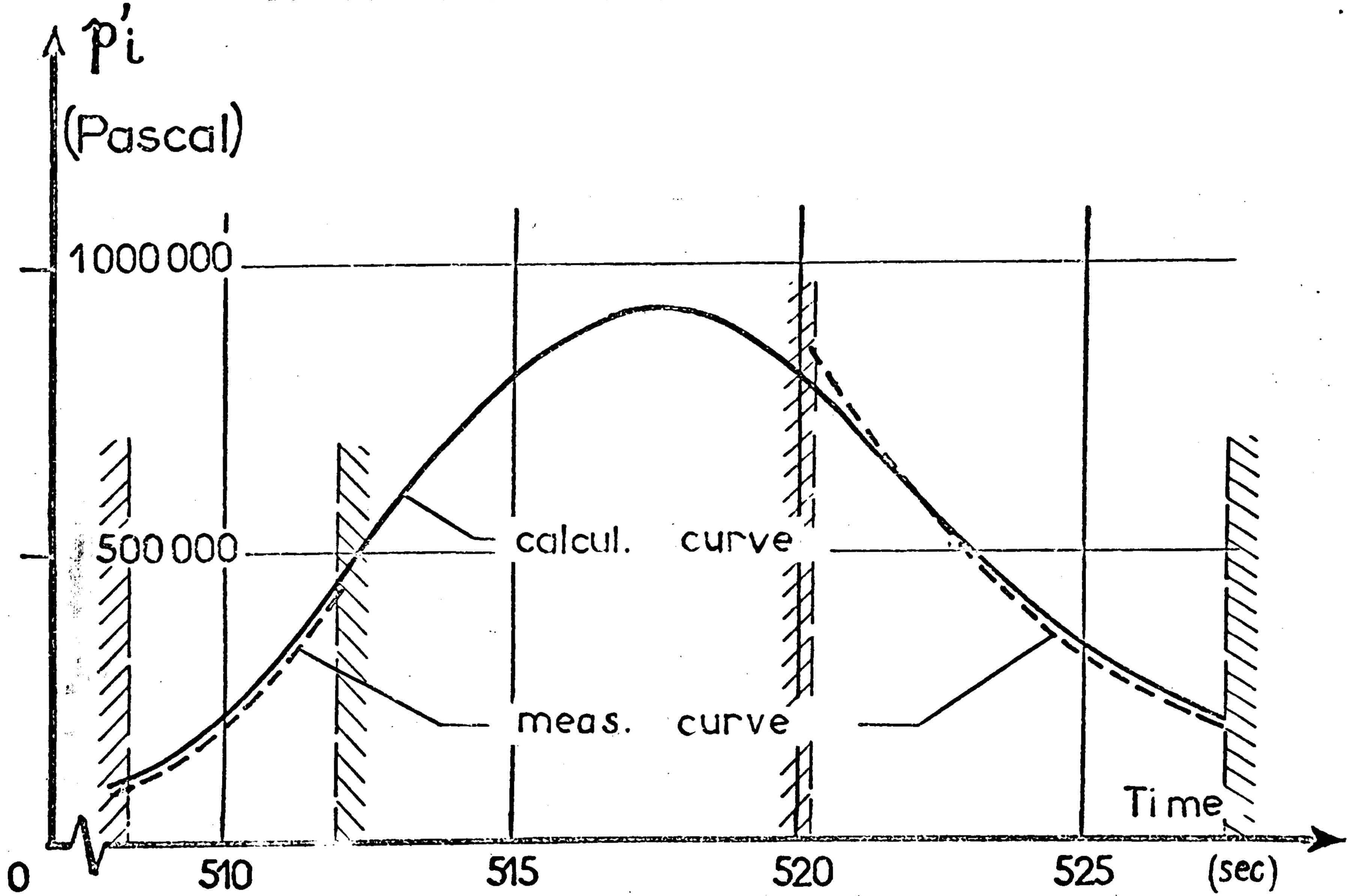

Fig.6 Probable Trajectory; Impact Pressure

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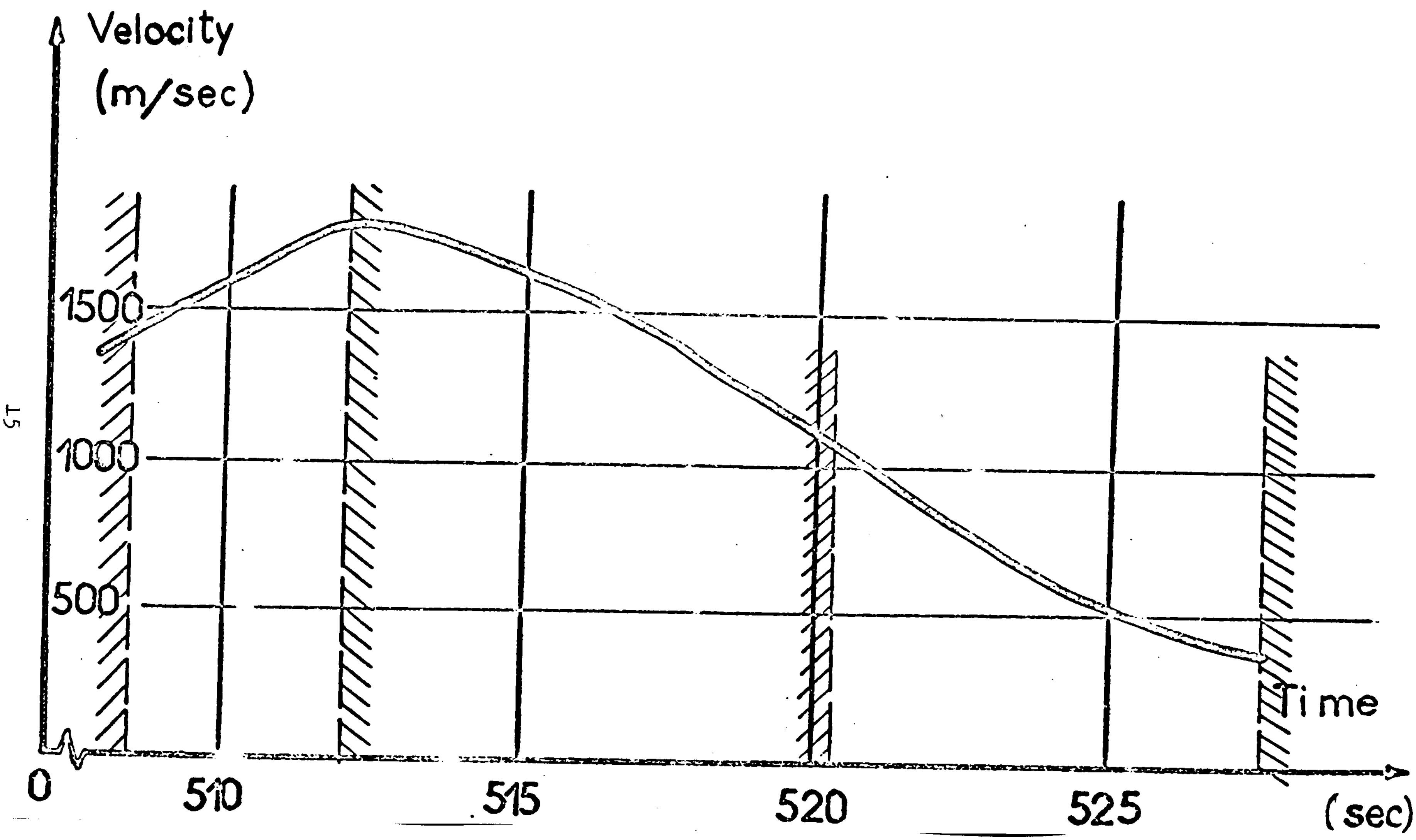

Fig.7 Probable Trajectory

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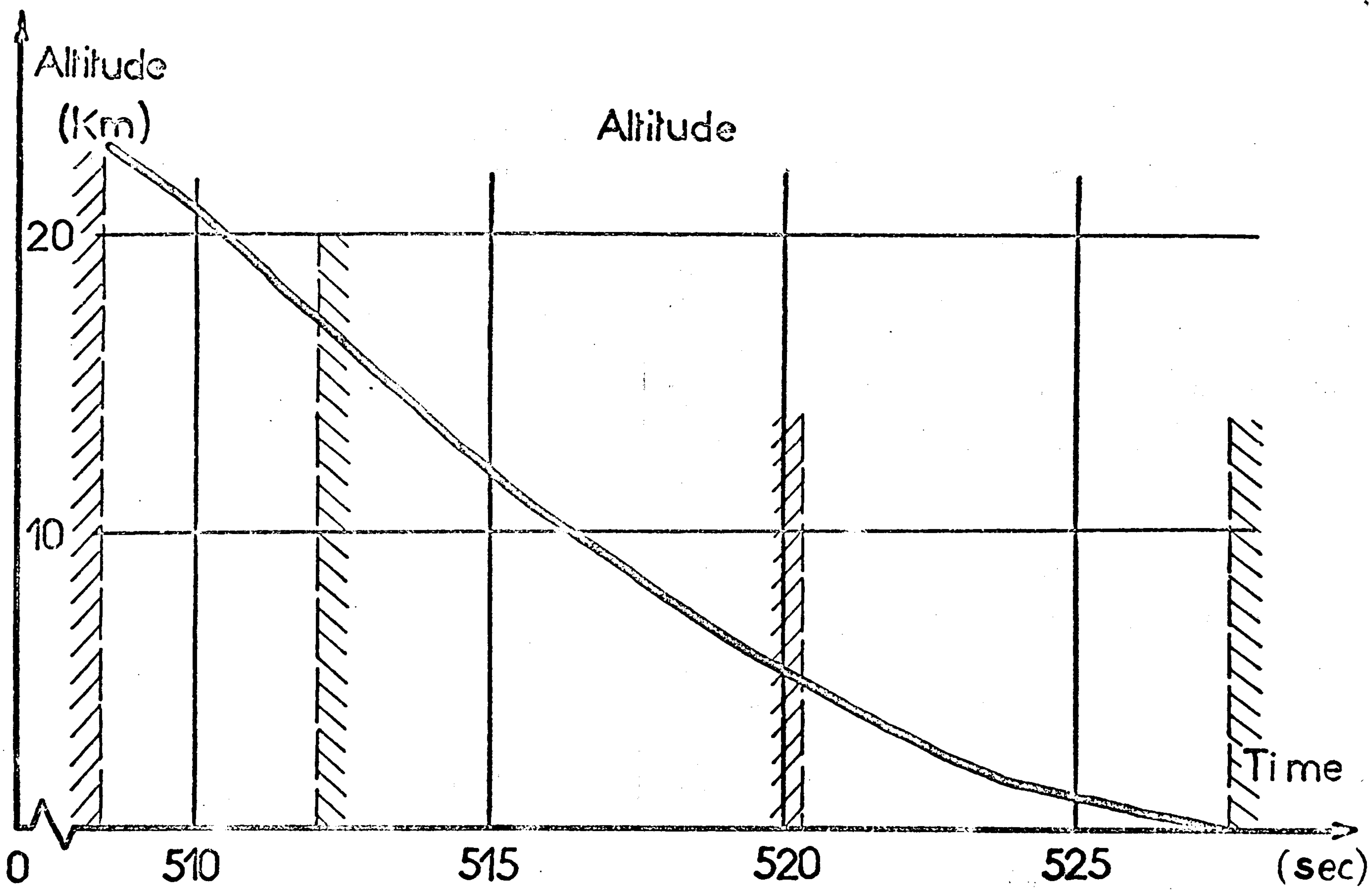

Fig.8 Probable Trajectory

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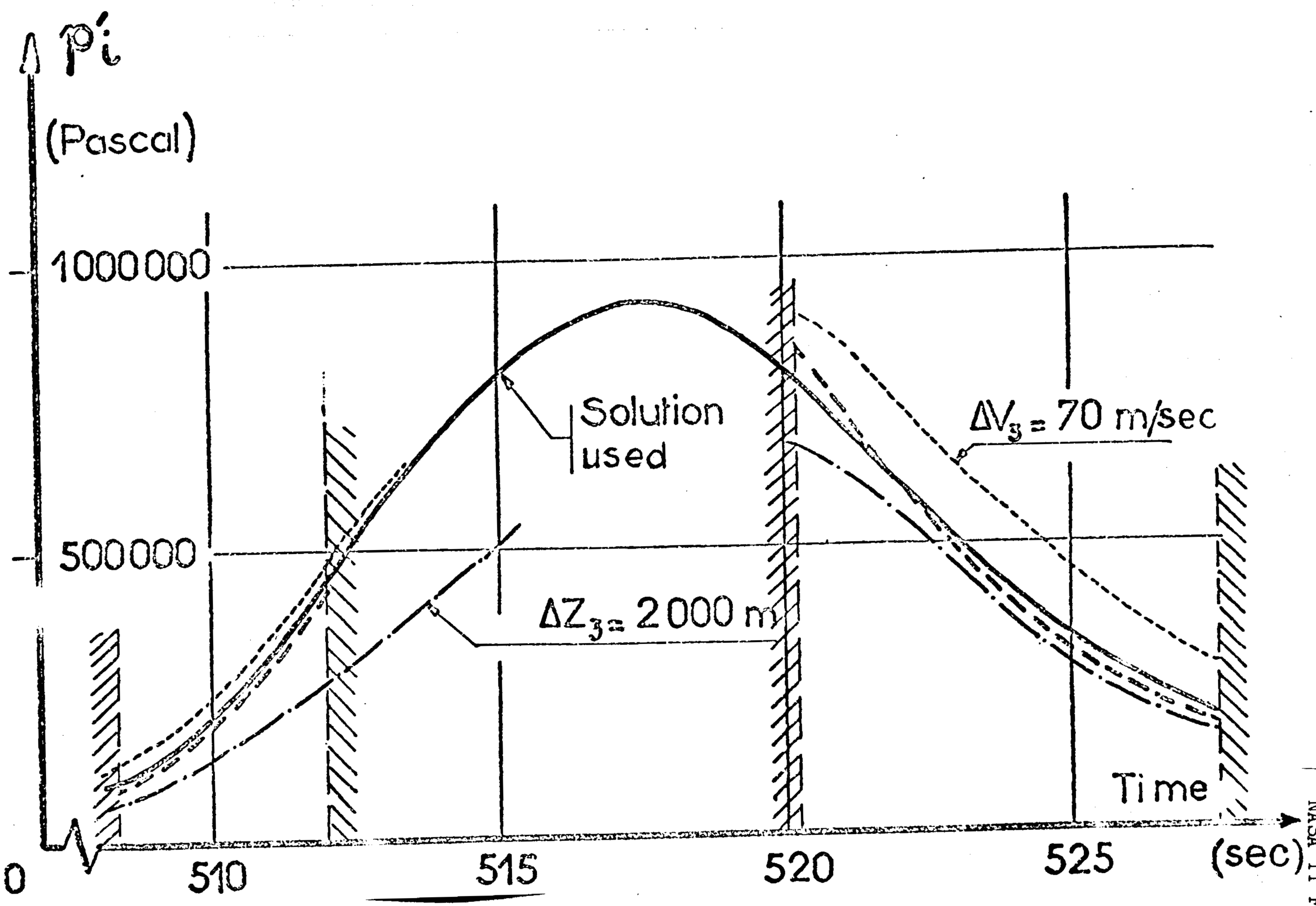

Fig.9 Impact Pressure; Relative Sensitivity to Z_3 and to V_3